<u>In our Time</u> Programme 30
Mathematics

Melvyn Bragg : Hello, Galileo wrote, "This grand book, the universe is written in the language of mathematics". It was said before Galileo, and has been said since, and in the last decade of the 20th century, it's being said again, most emphatically. So how important is maths in relation to other sciences at the end of the 20th century? What insight can it give us into the origins of life and the functioning of our brains? And what does it mean to say that mathematics has become more visual?

Joining me is Ian Stewart who's Professor of Mathematics, Gresham Professor of Geometry at the University of Warwick and on of the country's most prolific popularisers of mathematics, having written or co-authored over 60 books on the subject. Also an active research mathematician, with over 120 published papers, his most recent book was "Life's Other Secret: the New Mathematics of the Living World", published last year. Brian Butterworth is Professor of Cognitive Neuroscience at University College, London. His book "The Mathematical Brain" has just been published. It looks at the way that the brain deals with numbers, and how this is influenced by cultural factors. Currently, he's conducting research in Britain, Italy and China on arithmetical processes in normal and brain-damaged adults and children.

Ian Stewart you quote Carl Freidrich Gauss, who you consider to be one of the greatest mathematicians, who said in the 18th century that "maths should be about notions, not notations". Can you enlarge on this?

Ian Stewart : Yeah, I think he, Gauss, was trying to counter the idea that what maths is about is doing calculations with symbols, algebra, and these very formal things. Gauss was saying "no, it's about ideas", and those ideas ideas can be very, very broad, and what we're seeing now is that, this point of view is unfolding in a way that nobody would have anticipated at the time Gauss said it, which is that mathematics in certain senses is pervading all sorts of areas you wouldn't expect it to turn up in.

Melvyn Bragg : Can I just stop you there for a second?

Ian Stewart : Yeah.

Melvyn Bragg : People listening, and myself, I'm listening, think that maths is about adding up, subtracting, long division and so on, and people are much more sophisticated than that, but there you go. So when you say "ideas" can you give us an example?

Ian Stewart : Okay, let's take for example the question of when a living creature grows, say a baby growing in the womb, it grows from an embryo, it turns into a foetus, it turns into a child, and there are a whole lot of processes going on in changes of shape, changes of form, changes of internal chemistry, all of these things. Mathematics has quite a lot to say about that kind of behaviour, that kind of process, and I think in the future it will have a lot more to say about that.

Melvyn Bragg : So that's the idea. . . that is an *idea* of the way life develops....

Ian Stewart : Yep.

Melvyn Bragg : ...rather than a simple adding up? You say that it's also becoming more visual, can you tell us what you mean by that?

Ian Stewart : Mathematicians have spent the last 50 years or so , trying to turn visual intuition, informal visual intuition, the feeling we have in our heads for the shapes of things, into a kind of logically precise way of thinking and things you can write down, and so the...instead of just using numbers and doing arithmetic, there's a kind of free flowing mathematics of form, but it is just as mathematical, it is just as logical. The invention of topology, which is rubber sheet geometry, the idea that er....um...a coffee cup is the same as a doughnut, is what standard cliche for topology is, because a coffee cup has one hole in it, it's the hole in the handle, where you put your finger through, an American style doughnut has a hole in it. You can make a Plasticine doughnut and deform it slowly into a coffee cup, and for topologists those two things are the same, and there's a whole entirely rigorous mathematics, based on that which captures the idea of continuous changes in form.

Melvyn Bragg : Brian Butterworth, d'you. . . what d'you have to say about this visual representation?

Brian Butterworth : It's certainly true that the. . . one of the key visual areas in the brain, the parietal lobe, is one of the areas...

Melvyn Bragg : The parietal lobe?

Brian Butterworth : The parietal lobe...

Melvyn Bragg : Where's that?

Brian Butterworth : . . . that's the bit just behind your ear.....

Melvyn Bragg : Which ear?

Brian Butterworth : . . . er both ears.

Melvyn Bragg : Oh right.

Brian Butterworth : But the key one for mathematics is the bit behind the left ear. Now this idea of space is, space relative to ourselves, as opposed to space - the final frontier, is dealt with by this part of the brain, and so is mathematics. So it's clear that there are intimate links in the brain between these two notions, and it might well be that, somehow, as mathematicians become more sophisticated, some of the ideas of space which they were previously unable to formalise, have now become better integrated into the symbolic systems that they use. I think the point that I'd want to stress is, taking Gauss's idea of a notion, is that at least some of our most basic notions, the notion of number, seems to be built into our parietal lobe when we're born, so that we get a flying start, we're born , if you will, with a kind of "start-up kit", which enables us to immediately see that here are three cups on the table, and there are more cups on this table that there are on that table where there are only two. We don't need to learn that. That's already in our brains.

Melvyn Bragg : I mean, are you comparing it with the language instinct?

Brian Butterworth : I am comparing it with the language instinct?

Melvyn Bragg : And you think it's as innate. . . you please tell me what *you* mean by that word, as the language instinct?

Brian Butterworth : Well, I think we actually have better evidence that the number instinct, if you will, is innate, and we know which specific bits of the brain there are. . .

Melvyn Bragg : Could you tell us? Can you give us some of the evidence?

Brian Butterworth : Er. . . yes. . . er. . well first of all...er infants are born, able to recognise the number of objects in a display. So if you keep....

Melvyn Bragg : At what age?

Brian Butterworth : Er. . the day they're born.

Melvyn Bragg : The day they're born?

Brian Butterworth : Yes, so experiments have been done on neonates, in the first week of life, you show them an array of say two objects, and you show them another array of a different two objects, and you keep showing them arrays of two objects, and they get bored with this. Then you show them three objects, and they start to take an interest again, and then you keep showing them three objects and they get bored, and you show them two objects again, and they gain interest. Now...

Melvyn Bragg : What does this prove?

Brian Butterworth : It proves that they are sensitive to the number of things in what they see, and a later age, possibly even when they're born, but we don't know this....

Melvyn Bragg : I mean, are they more sensitive than say, seeing a little puppy wag its tail? Is it a different sort of sensitivity? Is it just a sensitivity to what's impinging on them? Or is it a particular numerical sensitivity?

Brian Butterworth : It's a particular numerical sensitivity. So you can change the object, so you can show them the same things and then change the object and they'll be sensitive to that. But they will be more sensitive if you actually change the number of things that they see. So the very young child. . . the very young baby, is actually more sensitive it seems to the number of things around him or her than to what those actually are.

Melvyn Bragg : So we are wired for numbers?

Brian Butterworth : **So we are wired for numbers.**

Melvyn Bragg : What does that say to you, Ian Stewart?

Ian Stewart : I mean, that's fascinating, and I though that the evidence that Brian presented in his book for that was quite compelling. This presumably. . . I don't know, I mean there's an evolutionary advantage in being able to spot rather early that something around you has changed, there is a new thing. . . has appeared, that . . . you know, you ought to be aware that something new has arrived that wasn't there before.

Melvyn Bragg : Two wolves rather than one, yeah.

Ian Stewart : Yeah, or one extra wolf compared to the two sheep that there were! So, maybe it comes from that, but actually that's a sort of fairy story really isn't it? But. . er yeah, this does seem to be very strongly wired in, and what's interesting here, is it's different from...I mean my understanding is a lot of our perceptual abilities are to some extent. . . the potential is wired in, but a certain amount of training is needed in the early weeks or months of life. There's this story about kittens. If kittens don't see vertical lines at a relevant time in the development of the visual part of their brain, then they can't perceive vertical lines very well, after that. Now this seems to be much more fundamental than that kind of thing, and that's very interesting.
Do you...? What I wonder is the extent to which.....just how fundamental to what I would call mathematics. I mean, you know, that's the start-up kit, but then you have a long way to go from there to the kind of thing that I'm assuming goes on inside my head, because at some point it tells me about it and I write it down.

Brian Butterworth : Yep, I'm not saying that every baby born able to grow up an Ian Stewart! I'm certainly not saying that. . .

Ian Stewart : (sniggers) Lucky them!

Brian Butterworth : (laughs) What the er. . . but it does give them, if you like, a head start when it comes to learning the kind of cultural tools that are provided by the culture into which they are born. Now clearly it's very different being born into the 20th century with topology, with non-standard analysis, with all kinds of interesting novel mathematics than it was being born, let's say in the time of Archimedes. So if you give Archimedes. . . if I wrote down for Archimedes, the greatest mathematician of antiquity, a simple quadratic equation, and asked him to solve it, he wouldn't be able to do it, because he just...for a start wouldn't know the notation. I mean, I'm sure he'd figure it out pretty quickly, but it means that, that the baby, to become an Ian Stewart, would need to be born into a culture which has those mathematical tools, and would have to acquire those mathematical tools, which means **a lot of hard work** and good education. I mean these tools come in a variety of forms, and some of the most obvious ones, you just don't think about. For example we all have number-words. Now there are some languages in which there are no number....

Melvyn Bragg : You mean, one, two three, four, simple as that?

Brian Butterworth : I mean, one two , three, four......

Melvyn Bragg : Yeah.

Brian Butterworth : ...so there are languages in which you don't have number words.

Melvyn Bragg : For instance?

Brian Butterworth : Errr. . well, there are languages in Australia which only have words for "one", " two", "three" and "many".

Melvyn Bragg : And there's Locke talking about the American Indians isn't there?

Brian Butterworth : Yes, and there's also. . . there are. . .

Melvyn Bragg : I think they could count up to twenty and then that was it.

Brian Butterworth : Well, John Locke, yes, the...

Melvyn Bragg : Yes.

Brian Butterworth : ...The English philosopher, was surprised that American's couldn't get above twenty, but these...

Melvyn Bragg : American Indians must stress at this point!

Brian Butterworth : Well we have to stress that they were American Indians, and they would use their own fingers to do counting and fingers of people who were nearby.

Melvyn Bragg : And body parts in Papua New Guinea. one tribe gets up to 33 by counting every body part. We can't describe on this programme the last three body part....

Brian Butterworth : Can't we! (laughs)

Melvyn Bragg : . . . that the male uses to get...... (laughter)!

Brian Butterworth : That's right. But actually our own. . . our own number words are derived from body parts as well. So...

Melvyn Bragg : What fascinated me....one of the things that fascinated me about your book, is the relationship between a part of the brain, and the actual fingers.

Brian Butterworth : Yeah.

Melvyn Bragg : That the fingers, and the mathematical part of the brain are related.

Brian Butterworth : Yep.

Melvyn Bragg : Now, is it...I mean that's just terrific, can you just tell people what's going on there?

Brian Butterworth : (laughs) Okay, well there's a circuit in the brain which involves the parietal lobe, and it. . .

Melvyn Bragg : Behind the ear.

Brian Butterworth : ....and it links up with a bit in front of the ear, called the"motor-strip", which actually does the fine planning of the motor movements, and one or two other motor areas in the brain. the control of our fingers is actually quite complicated, but the top. . . the topmost decision area seems to be in the parietal lobe, and it may well be, and this is one of the speculations I advance in my book, that what's happened in the course of the development of the individual child, and also in the evolution of our culture, is that the fingers gain additional meanings. They gain the meanings of the numbers that are associated with them, when a child learns to count on his fingers, or to do arithmetic on his fingers, and so you get what's called "activity dependent brain changes", so that the representation in the brain of the fingers expands, and includes these numerical representations as well. So ask a very odd question for a neuroscientist, I say "Why is it that numerical representations are in the parietal lobe, rather than in the temporal lobe which is just below the ear, where most of our other knowledge is?". So, it seems to me that might be one of the answers.

Melvyn Bragg : And that takes us to St Bead, the venerable Bead's finger counter, but it also took me on a fantasy about this relationship has always been talked about between mathematics and music , and that Mozart could have been a wonderful mathematician, so we're told. I wondered whether, sort of, the fingers which operate the piano (laughter) are related to the feat! This is...we are entering into the sort of...we're getting way away from you two are....really know about. Ian Stewart, you're talking about "visual mathematics", and you say that Chaos theory has emerged from mathematicians using *visual* geometric maths. Could you tell us how that's happened? I mean we're all fascinated by Chaos theory, and I've been told often enough by people that we don't understand it well enough! So can you try to make us understand it from this point of view?

Ian Stewart : If you look at where it came from historically, people were trying to solve let's say the three body problem in astronomy . You've got the Earth the Moon and the Sun, and you want to know how they move under gravity, and the answer is "it's very complicated", and the way they were trying to solve it was the traditional way, write down the equations, solve them, very much think of it as a symbol manipulation problem, and you've got to calculate lot's of numbers, and you've got to get some formulas for the answer. Now it turns out there isn't a formula for the answer, and the reason emerged. . . er the best explanation why there isn't a formula, came when people started to think about that problem in a geometric way. It's a bit abstract but you think about it as the flow of time in a system like that, things move, and you kind of map out, not just one path that the planets might move on, but all possible paths, and then what you have is a kind of high-dimensional fluid flowing in a space, and you say "how does that fluid move?" and the answer is it's a bit like when you're making bread, you stretch it, fold it, knead it, and this kneading process, stretch and fold, pull things apart and then fold them back in again, is where Chaos comes from. Because if you're stretching things apart, points that start very close together, move further and further away , if you're folding things back in they start to become independent of each other, this is The Butterfly Effect of Chaos theory. This is the "very small change at some stage, can have a huge effect later on . If you had two little grains of dust in your bread, and you keep kneading it, after a while they are in completely different parts of the bread and there moving around in completely different ways. This is why we knead bread, it's to mix it all up, and so it's a geometric picture of the mixing, and that really is the key to Chaos.

Melvyn Bragg : One of the things in your book, that interested me is that you're challenging the idea that DNA is the answer to the secret of life, is the secret of life. You give it all due credit, and you know far more about it than I'll ever know, but what you're saying is that behind DNA, the deeper structure is mathematical, and not biochemical. Am I right? And if so can you just give us some...

Ian Stewart : That's the big message, the , as you say I give DNA due...DNA has to be given huge amounts of credit. The science of DNA is extremely important, you won't understand life on this planet without it. But the question is "What is it that it plugs into? What's the background into which it goes?" , and I think the current view is much too naive. It kind of....it sees it as some sort of blueprint, some sort of map of the organism . People will talk about "finding the bit of DNA that controls the growth of an ear" or something like this, and it isn't really like that. What DNA is doing. . . it's embedded in a much more complex process. But of course the process runs on the basis of the laws of physics and chemistry. If I push an elephant of a cliff, its DNA does not have to specify that it will fall under gravity, but the law of gravity makes that happen. there's all sorts of processes going on which biologists interested in DNA tend to think "Oh these are just background, these are just default, these are going to happen anyway, we don't need to worry about them", but the problems is, what the DNA's role is, is to modify those processes, to kind of drive them through the developmental landscape. It's like if you're trying to drive a car, its no good just having a map, you have to actually know how to turn the steering wheel, you have to know which roads coming up, you have to have a lot of extra things that aren't really "in the map", and the way all that stuff works is mathematical, not in the sense that nature is doing mathematics, in the sense that human beings have to do mathematics to understand what it is nature is up to.

Melvyn Bragg : Is this useful? You, you study, Brian Butterworth, you're studying the brain and you're working with stroke victims and accident victims, and as I understand it, please correct me if I'm wrong, the great advantage of that, I mean unhappy circumstances, but if somebody suddenly can't count, you can work out which part of the brain has been damaged which causes them not to count, therefore you can discover which part of the brain controls counting. How is mathematics. . . er what Ian Stewart has been talking about, the mathematics behind DNA, is that relevant to what you're doing?

Brian Butterworth : I think what Ian has to say about mathematics is extremely interesting and about it's role in DNA, but also he had a very interesting metaphor in his book, which is the CD metaphor, "It's not enough to have a CD, you also have to have a CD layer", and that in order to understand how our brains come to be how the way they are, its not enough just to have the CD that contains, if you like, the program, you also need to have all the other stuff, which turns this program into music, if you will. So we need to understand two things when we're trying to find out which bit of the brain does mathematics. We need to find out, first of all which bits we can knock out without affecting mathematics. So if we knock out bits that aren't the parietal lobe, on the whole mathematics will remain intact. We've got quite a few patients now, including one rather prodigious calculator, whose parietal lobe is intact but everything else seems to be shot to pieces, and also the other way round. Which bits you knock out which will affect mathematics. So that tells us where in the adult brain numbers are primarily processed.
But we also want to know it comes about that, that bit of brain is the relevant bit, and here knowing about the DNA is quite important. Because we now know that there are populations of people with genetic abnormalities, that is abnormalities of their DNA, who have particular difficulties with numbers, and so we can actually get to. . . we're getting to look at the CD, to find out which bits of it seem to be faulty. But then we have to see, of course, this faulty CD creates the music in the CD player and why this music becomes the way it does. So I think this is a very interesting idea that many people in neuroscience are beginning to get a grip on.

Melvyn Bragg : As a species, both of you really, do you think we could function without numbers? Do you think we could function with just language but not numbers? Ian Stewart first.

Ian Stewart : I don't... well, I'm sure we could function. I'm suspect our linguistic abilities would push us in the direction of inventing...if numbers didn't exist we would probably invent them.

Melvyn Bragg : I mean there's a massive part of Brian's book which I pinched on my trail (laughter) saying he reads...in the course of reading a newspaper , and going through a day, there's about 15, 000 numbers you read everyday or something like that.

Ian Stewart : You see, we can't read without a huge amount of visual processing going on in the brain which can handle what is actually enormously sophisticated mathematics , as the computer engineers who try to get computers to read written symbols know , there's some very heavy processing...now a brain that can actually recognise the letter "A" on the printed page can probably do arithmetic very easily in comparison. Okay the connections may be in the wrong place and so forth. But in a sense, the sophistication of the program that's running in the brain, to recognise the letter "A" is much greater than 2+2 = 4.

Melvyn Bragg : Is there any way. . . it's ridiculous, we've only got about 7 minutes left , but is there any way that you two can help people. . not help, I don't mean help, I don't think people need help, I hope they're enjoying themselves, but you've talked to people who er. . Ian, both who have had strokes and accidents, and people in societies which...they conduct in their societies by only being able to count to 33, as it were, and your, Ian Stewart, that's Brian Butterworth, you Ian Stewart are talking about mathematics which is in a completely different realm from the mathematics that normally educated people, like myself know about . Is there any way in which you two are feeding each other?

Ian Stewart : I think so. I mean in the simple...I read Brian's book, got a lot out of it, and that's changed my thinking on some of these things.

Melvyn Bragg : In what ways?

Ian Stewart : Well the idea...he's addressing the inside story, I'm sitting there thinking "What can mathematics tell us about how the universe outside works?" and he, in a sense, is saying, "How can our knowledge of the structure of the material...the way that our brains are actually put together as bits of machinery, tell us about what it's like to be inside a mathematical mind, and how does that work?". It's kind of the converse problem, and therefore these points of view, I think very much, they feed off each other. /Now the two of us can't do a huge amount, but science doesn't work that way. but think of scientific communities across the world putting the inside and the outside together in that way, at the very least we should get a much better understanding of how a lot of things work , because I think the problem, until fairly recently, science has tended to be quite compartmentalised. You miss big parts of the picture and very clearly one of the big changes over the last 10 or 15 years has been that a lot of the boundaries between disciplines are dissolv....mathematicians can come in and meddle with biology, and the biologists don't like it, but every so often they say "Oh that's interesting, maybe you've got something there. We'll go away and think about it".

Melvyn Bragg : How important to you, Brian Butterworth, are the conclusions you've drawn from the discovery of discalc...dis-calc-ulia , and can you just explain to people, who can. . . who don't understand it, I can't even pronounce it properly, what we're talking about there?

Brian Butterworth : There are some people who are born unable to grasp simple numerical notions. I mean they are unable to see that there are 3 cups on the table. They can, as Ian suggested, learn a language which allows them to conduct a procedure of counting those cups, but unlike me for example, and the vast majority of people, they can't just see that there are 3 cups on the table. These people don't end up....

Melvyn Bragg : Is it a bit like dyslexia?

Brian Butterworth : It's actually more profound than dyslexia in the following sense: that, for up until the last 100 years, the wasn't widespread literacy in the world. There wasn't universal literacy. Literacy wasn't important evolutionarily, but numbers have always been important evolutionarily, even lions can use numbers up to a certain extent. Lions are meant to be quite stupid. But being able to read, and therefore disorders of reading, is a result of not being able to put together bits of brain that were designed for other purposes. but being discalculic is a bit like being colour blind. There's a special bit of the brain for seeing the world in a particular way. For seeing colour, it's the colour parts of the brain, "V4" it's called, it's for seeing the world in colour, we do it automatically, we can't help it. The parietal lobes give us the ability to see the world in numbers, we can't help it, it's automatic.

Melvyn Bragg : But to come back to my very clumsy question, I do, but the two of you are here together on this programme, and here we are talking, but is it possible, between you, have an understanding of mathematical concepts, which is what you're talking about Ian Stewart, brilliantly I think, , and at the same time have a very limited notion of numbers?

Ian Stewart : There's the question! This is the big...this is back to what we were saying a little bit earlier about "how do you get from your start-up kit to . . . what I suspect is not as sophisticated as what it seems to be . I'm not convinced that when a leading research mathematician sits and does something absolutely amazing in 17 -dimensional algebraic topology, that they're as far floating in the heavens above the ordinary mortals as we tend to assume. Because you've got to get there with a fairly ordinary human brain. There's, as Brian says in his book, there's a certain amount of training and other things go in. I think there is at least some sort of predisposition to be. . . to respond to training in that sort of direction. I'm not sure. I don't think you could take anybody and teach them this kind of mathematics.

Melvyn Bragg : Brian Butterworth?

Brian Butterworth : Well, I think you can teach most people, as much mathematics as they would like to learn. Discalculics being an exception. What I think is critical here, is how much time they're willing to put into it, and how well they're taught . Now one of the things we don't know, is whether, and this is a straightforwardly empirical question, is whether people who are born without this number sense, are able, for example, to do topology, which doesn't depend so critically on numbers, I mean there are some numerical aspects to topological theorems, but the question is could they get a sense of geometry of topology, which doesn't depend critically on numbers. The answer is , "we don't know yet".

Melvyn Bragg : Do you think you could stimulate parts of the brain to be better at mathematics?

Brian Butterworth : Well you do stimulate parts of the brain to be better at mathematics by training it. I mean we know that brain areas that are worked heavily tend to become more densely interconnected. They tend to get bigger. So if you give people a lot of maths to do, then the maths parts of their brain are going to get bigger and better.

Melvyn Bragg : This is a ludicrous question to throw at you with a minute to go, Ian Stewart (Brian laughs), but you agree with Galileo don't you? That the universe is written in the language of mathematics?

Ian Stewart : I'm a little bit ambivalent on it. I guess about 95 percent of me agrees with Galileo and there's always that other 5 percent that sits there saying "Yes but you're biased!" (Brian sniggers) . Not just me as a mathematician, but our whole culture you know....

Melvyn Bragg : You're not only agreeing with Galileo, you're agreeing Pythagoras, you're agreeing with Newton, quite a good team you've got out there! They could win the league!

Ian Stewart : There's something going on in the universe on a very deep level that's mathematical, but I'm not sure it's the equations the physicists haul out and say "This is fundamental". I think it's something else.

Melvyn Bragg : Well thank you very much. There's lots more to talk about but maybe we *three* will meet again. Thank you Ian Stewart. thank you Brain Butterworth, and thank you for listening.